Intrinsic Insertion Loss of a Mismatched Microwave Network

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Summary—**A brief review is presented of the absolute minimum loss of a two-port, mismatched lossy microwave network. This minimum loss, called the "intrinsic insertion loss" can be attained by adding suitable susceptances at the ports that will yield simultaneous bilateral network match. A measurement procedure is described and convenient graphs are presented.**

Introduction

Lossy microwave networks which are matched in impedance have been analyzed extensively. On the other hand, mismatched lossy networks are more difficult to analyze since the apparent and real losses depend upon the degree of mismatch as well as on the terminating impedances. A brief review is presented of the absolute minimum loss of a mismatched lossy network. This is followed by a measurement procedure to determine the loss and an example.

A general two-port lossy network can be completely identified by a voltage scattering matrix of the form[1,2]

$$S = \left\| \begin{matrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{matrix} \right\|, \tag{1}$$

where S_{11} and S_{22} are the reflection coefficients at the respective ports, S_{12} is the transmission coefficient of the network, and for reciprocity $S_{12}=S_{21}$. These coefficients can be measured by using the method described by Deschamps[3] or Storer, Sheingold anrd Stein.[4]

When a mismatched lossy netwok is terminated in a matched load, two different losses can be defined, depending upon the choice of input power, either incident or net. If the incident power is chosen, the loss is

$$\alpha_{S_{12}} = -10 \log_{10} |S_{12}|^2 \text{ db}. \tag{2}$$

$\alpha_{S_{12}}$ is sometimes called the insertion loss in a matched system,[5] i.e., both oscillator and load are matched.

However, loss is related to net input power,

$$\alpha_{\text{net}} = \alpha_{S_{12}} - \alpha_r = -10 \log_{10} \frac{|S_{12}|^2}{1 - |S_{11}|^2} \text{db}, \tag{3}$$

where α_r is the input reflection loss. Since α_{net} represents the amount of heat dissipated in the network relative to the net input power, α_{net} is sometimes called the transmission or circuit efficiency.[6] Note that $\alpha_{\text{net}} \leqq \alpha_{S_{12}}$.

If now a suitable susceptance is added to the input port to cancel the effect of S_{11} per se, thus providing a unilateral network match, the combined effective S_{12} will increase in magnitude such that the insertion loss of the combined network in a matched system will become equal to the initial α_{net}. It then follows from (3) that for a general mismatched lossy network where $|S_{11}| \neq |S_{22}|$, the value of α_{net} will depend upon the direction of power flow through the network.

Experimentally, α_{net} can be measured easily by the method described by Cullen[6] ($\alpha_{\text{net}} = -10 \log_{10} \eta$), Beatty[7] ($\alpha_{\text{net}} = A_D$), Deschamps[3] or Beatty and MacPherson.[8] These methods use an adjustable short circuit on the input port of the mismatched lossy network while the network admittances are measured at the output. By plotting these admittances as reflection coefficients on a polar diagram,

$$\alpha_{\text{net}} = 10 \log_{10} (1/R) \text{ db}, \tag{4}$$

where R is the radius of the locus in units of the reflection coefficient. For a mismatched network this circular locus is not centered on the polar diagram. The quantity α_r can be determined from a separate measurement using a matched load on the output port. In Beatty's article, $A_T = \alpha_{S_{12}}$ and $A_R = \alpha_r$.

Intrinsic Insertion Loss

Although the insertion loss due to a mismatched lossy network can be reduced by the addition of a suitable susceptance at the input port to achieve a unilateral match, it is possible to reduce the insertion loss even further by placing appropriate reactive elements at either port of the network that would provide a bilateral network match.[9,10] Under the bilaterally matched condition, the loss will be the absolute minimum attainable and will be called "intrinsic insertion loss."

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1 R. H. Dicke, "Principles of Microwave Circuits," edited by C. G. Montgomery, R. H. Dicke, and E. M. Purcell, M.I.T. Radiation Lab. Ser., vol. 8, McGraw-Hill Book Co., Inc., N. Y., pp. 146–150; 1948.

2 N. Marcuvitz, "Waveguide Handbook," M.I.T. Radiation Lab. Ser., vol. 10, McGraw-Hill Book Co., Inc., N. Y., p. 107; 1951.

3 G. A. Deschamps, "Determination of reflection coefficients and insertion loss of a wave-guide junction," *Jour. Appl. Phys.*, vol. 24, pp. 1046–1050; August, 1953.

4 J. E. Storer, L. S. Sheingold and S. Stein, "A simple graphical analysis of a two-port waveguide junction," Proc. I.R.E., vol. 41, pp. 1004–1013; August, 1953.

5 R. N. Griesheimer, "Technique of Microwave Measurements," edited by C. G. Montgomery, M.I.T. Radiation Lab. Ser., vol. 11, ch. 11, McGraw-Hill Book Company, Inc., N. Y., p. 680; 1947.

6 A. L. Cullen, "The measurement of transmission efficiency at microwave frequencies," *Wireless Eng.*, vol. 26, p. 255; August, 1949.

7 R. W. Beatty, "Determination of attenuation from impedance measurements," Proc. I.R.E., vol. 38, p. 895; August, 1950.

8 R. W. Beatty and A. C. MacPherson, "Mismatch errors in microwave power measurements," Proc. I.R.E., vol. 41, p. 1118; September, 1953.

9 For example, S. Roberts, "Conjugate-image impedances," Proc. I.R.E., vol. 34, pp. 198P–204P; April, 1946.

10 H. A. Wheeler and D. Dettinger, "Measuring the Efficiency of a Superhetrodyne Converter by the Input Impedance Circle Diagram," Wheeler Monographs, vol. 1, no. 9, Wheeler Labs., Great Neck, N.Y.; 1953.

Since, for a general lossy network, an external susceptance that will match the scattering coefficient S_{11} per se will not match S_{22}, the problem of bilateral matching is rather difficult to solve using the concept of scattering coefficients or even using the other equivalent circuits of lossy networks such as T or π networks. By utilizing a new equivalent circuit for a mismatched lossy network introduced by Wheeler,[10,11] the bilateral matching procedure as well as the physical significance of intrinsic insertion loss becomes readily apparent. In this paper a measurement procedure is described that will give the intrinsic loss and other constants of the new equivalent circuit. An example is presented for illustrative purposes.

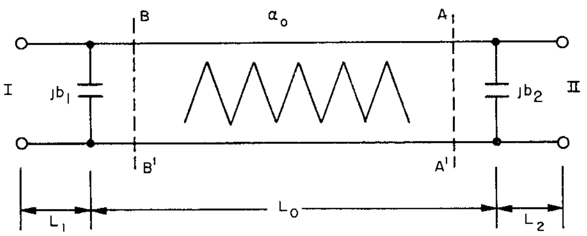

Fig. 1—Equivalent circuit of a lossy two-port network.

The new equivalent circuit, which has six independent parameters as found in either a T or π network, consists of two shunt susceptances and a matched attenuator as shown in Fig. 1. The locations and magnitudes of the two susceptances yield four parameters. The other two parameters are the transmission loss and transmission phase of the matched attenuator. If suitable susceptances are connected at the ports to cancel the effects of b_1 and b_2, the network will be bilaterally matched and the insertion loss will be a minimum equal to α_0, the intrinsic insertion loss. The equivalent circuit in Fig. 1 with a matched load on port II illustrates that α_{net} may be greater than α_0 due to the attenuated reflection loss from susceptance b_2. For an arbitrary network the values of the elements in the equivalent circuit will be frequency sensitive.

Measurement of Equivalent Circuit Constants

A relatively simple experimental method of determining the constants of the equivalent circuit consists of measuring the input admittance of the lossy network when terminated in a variable reactance, such as a sliding short circuit. Referring to Fig. 1, the short circuit is placed on port II. The VSWR at AA' is infinite. At BB', however, the VSWR is finite, and if the position of the short circuit is moved, the locus of the admittance at BB' when plotted on a Smith chart will be a circle concentric with the center. Thus, the VSWR at BB', $S_{BB'}$, will be a constant for all positions of the short circuit. The attenuation α_0 is related to this VSWR by the equation[12]

$$\alpha_0 = 8.686 \coth^{-1} S_{BB'} \text{ db.} \tag{5}$$

With the addition of susceptance b_1, the locus becomes off center, yielding a maximum and minimum VSWR. The intrinsic insertion loss α_0 can be calculated[13] from

$$\alpha_0 = 8.686 \coth^{-1} \sqrt{S_{max} S_{min}} \text{ db} \tag{6}$$

if the input admittance locus *encloses* the center of the Smith chart. If the locus does *not* enclose the center, however, α_0 may be obtained from the equation

$$\alpha_0 = 8.686 \coth^{-1} \sqrt{S_{max}/S_{min}} \text{ db.} \tag{7}$$

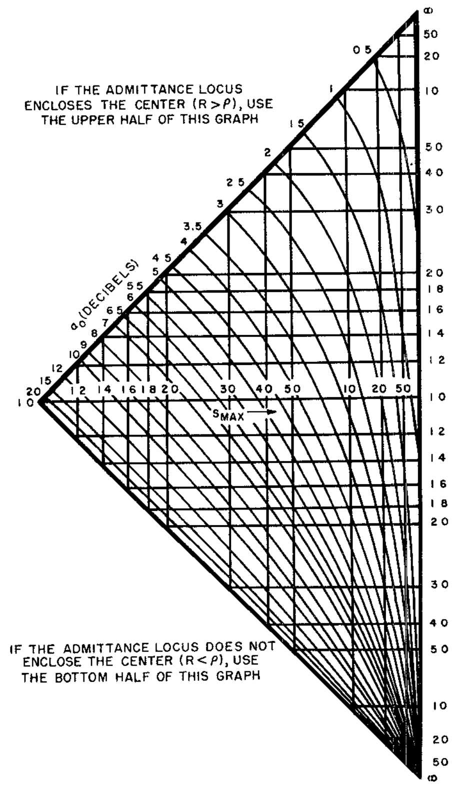

Fig. 2—Intrinsic insertion loss α_0 as a function of S_{max} and S_{min}.

These relationships between α_0, S_{max} and S_{min} are illustrated in Fig. 2.

The choice between formulas (6) and (7) can be eliminated if the input admittances are plotted as reflection coefficients on a polar diagram. The intrinsic insertion loss is given by

$$\alpha_0 = 10 \log_{10} \frac{\sqrt{(1+R)^2 - \rho^2} + \sqrt{(1-R)^2 - \rho^2}}{\sqrt{(1+R)^2 - \rho^2} - \sqrt{(1-R)^2 - \rho^2}} \text{ db,} \tag{8}$$

[11] H. A. Wheeler, "The Transmission Efficiency of Linear Networks and Frequency Changers," Wheeler Monographs, vol. 1, no. 10, Wheeler Labs., Great Neck, N.Y.; 1953.

[12] R. King, "Transmission-line theory and its application," *Jour. Appl. Phys.*, vol. 14, pp. 577–600; November, 1943.

[13] R. M. Fano and A. W. Lawson, "Microwave Transmission Circuits," edited by G. L. Ragan, M.I.T. Radiation Lab. Ser., vol. 9, McGraw-Hill Book Company, Inc., N.Y., ch. 9, pp. 553–554; 1948.

where R is the radius of the circular locus and ρ is the distance between the centers of the circular locus and polar diagram in units of the reflection coefficient. This formula is plotted in parametric form in Fig. 3. Note that for a given value of α_0, R must decrease when ρ increases. Eq. (8) reduces to (4) when $\rho=0$.

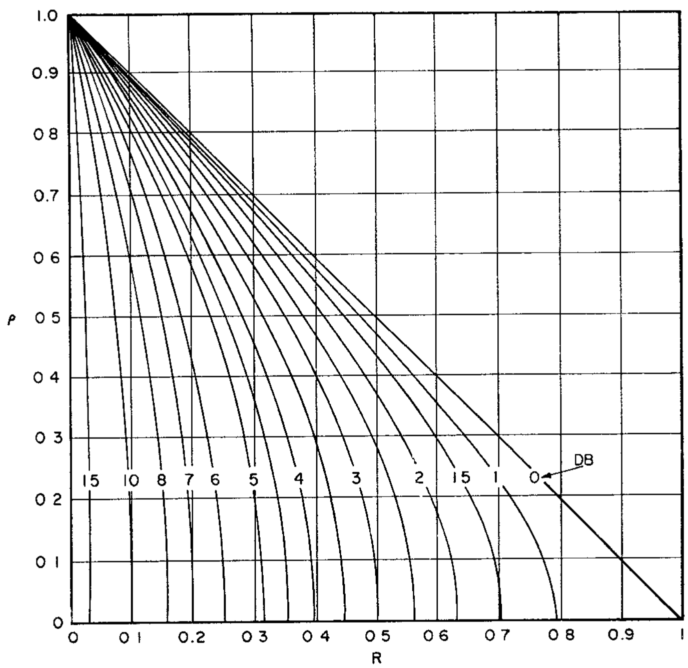

Fig. 3—Intrinsic insertion loss α_0 as a function of ρ and R.

The magnitude of the normalized susceptance b_1 can be determined from S_{max} and S_{min}, using the VSWR, S_1, which would be produced by b_1 on a matched line.[13] Accordingly,

$$S_1 = \sqrt{S_{max}/S_{min}} \tag{9}$$

when the admittance locus encloses the center. The equation

$$S_1 = \sqrt{S_{max}S_{min}} \tag{10}$$

is used if the locus does not enclose the center. These two equations are plotted in Fig. 4. The magnitude of the normalized susceptance b_1 can be calculated from

$$|b_1| = \sqrt{S_1} - 1/\sqrt{S_1}. \tag{11}$$

The choice of formulas (9) or (10) can be eliminated if the input admittances are plotted as reflection coefficients on a polar diagram. Then

$$S_1 = \sqrt{\frac{(1+\rho)^2 - R^2}{(1-\rho)^2 - R^2}}\,. \tag{12}$$

Eq. (12) is plotted in Fig. 5.

Similarly, b_2 can be determined by reversing the network ports and repeating the measurement. The line lengths L_0, L_1 and L_2 can be determined by using a

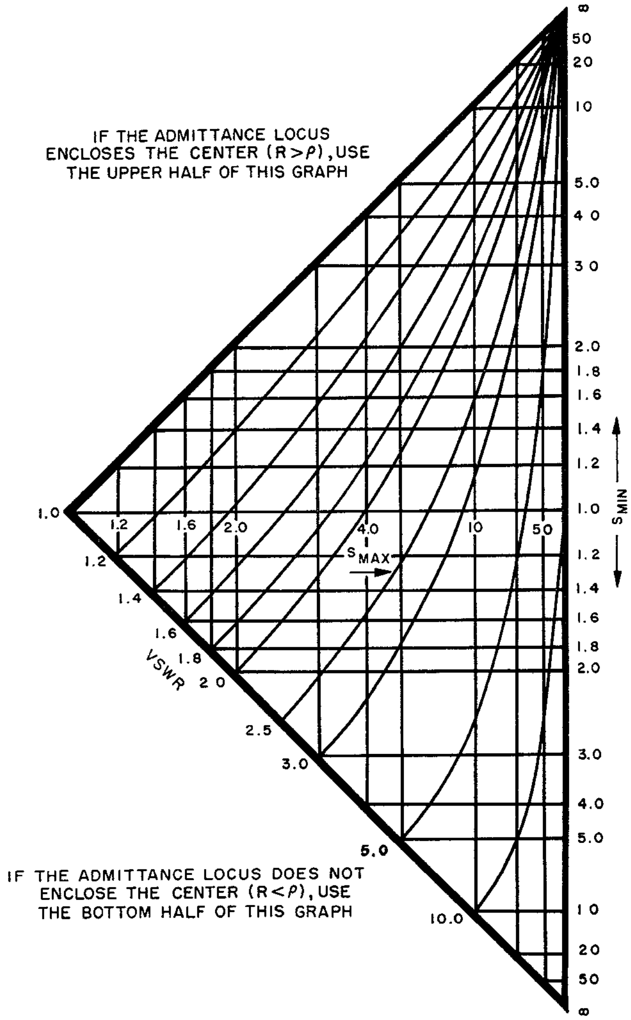

Fig. 4—VSWR as a function of S_{max} and S_{min}.

known short circuit reference plane and solving for the circuit constants from conventional transmission line circuit analysis.

An alternate method of determining the circuit constants follows in part that described by Deschamps[3] or Storer, Sheingold and Stein.[4] A sliding short circuit of known positions is placed on the output port and the

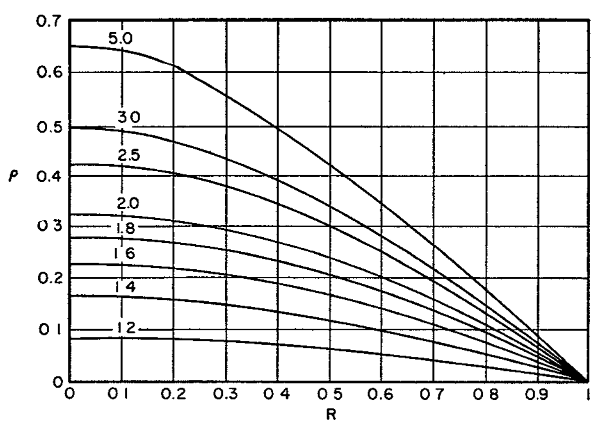

Fig. 5—VSWR as a function of ρ and R.

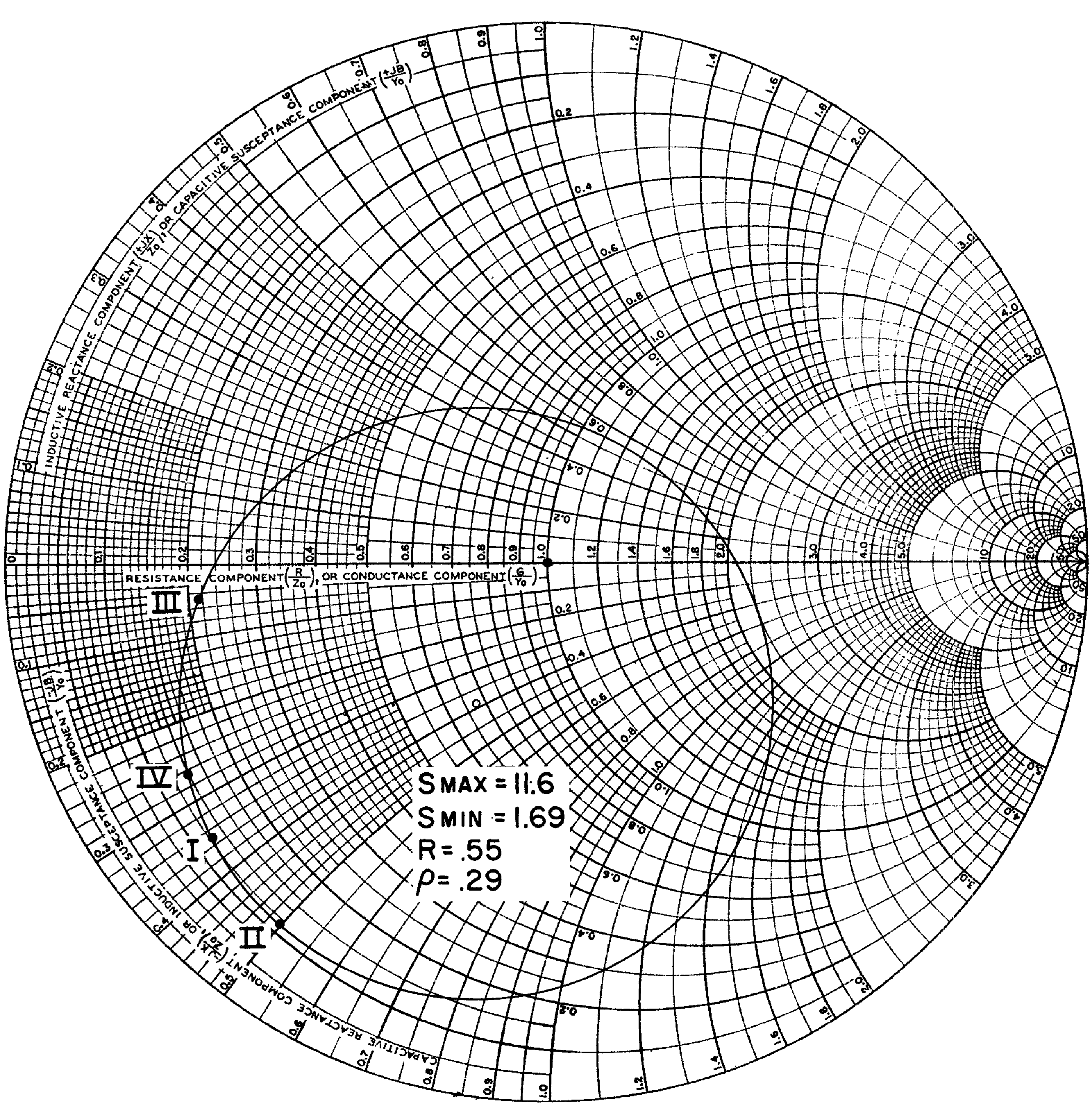

Fig. 6—Admittance locus of a mismatched lossy network.

input admittances are measured. Inasmuch as the susceptances b_1 and b_2 must be determined, a Smith chart presentation is required. The quantities b_1 and L_1 can be calculated by rotating the locus to such a position where it will be bound by reciprocal conductance lines. The locus and admittance points are then transformed to the center of the chart by subtracting the effect due to susceptance b_1. Referring to Fig. 1, this is now the admittance locus at plane BB'. After cancelling the effect due to α_0, the "iconocenter"[3,4] is located. This will yield b_2, L_0 and L_2.

Example

The significance of the different insertion losses can be illustrated by the following example. With a variable short circuit placed on the output port, the input admittance of a particular mismatched lossy network is plotted in Fig. 6. Admittance point I is obtained when the reference plane of the short is at port II. The other admittances are obtained when the short is moved successively away from the network in $\frac{1}{8}$-wavelength intervals. Using the method described in the literature,[3,4] the following scattering coefficients are obtained:

$$\begin{aligned} |S_{11}| &= 0.68 \\ |S_{22}| &= 0.78 \\ |S_{12}| &= 0.46. \end{aligned} \tag{13}$$

In accordance with (2),

$$\alpha_{S_{12}} = 6.7 \text{ db}. \tag{14}$$

If power is applied to port I, (3) yields

$$\begin{aligned} (\alpha_{\text{net}})_{\text{I}\to\text{II}} &= -10 \log_{10} [0.46^2/(1-0.68^2)] \\ &= 4.0 \text{ db}, \end{aligned} \tag{15}$$

or, if power is applied to port II,

$$\begin{aligned} (\alpha_{\text{net}})_{\text{II}\to\text{I}} &= -10 \log_{10} [0.46^2/(1-0.78^2)] \\ &= 2.6 \text{ db}. \end{aligned} \tag{16}$$

From the admittance locus in Fig. 6, the following is obtained:

$$\begin{aligned} S_{\max} &= 11.6 \\ S_{\min} &= 1.69 \\ R &= 0.55 \\ \rho &= 0.29. \end{aligned} \tag{17}$$

The intrinsic loss for this network is

$$\alpha_0 = 2.0 \text{ db}. \tag{18}$$

Since $S_1 = 2.62$, (11) yields

$$b_1 = 1.0. \tag{19}$$

The remaining circuit constants are

$$\begin{aligned} b_2 &= 2.0 \\ L_1 &= \lambda/4 \\ L_0 &= \lambda \\ L_2 &= \lambda/4. \end{aligned} \tag{20}$$

Discussion

It should be pointed out that while an actual microwave network may have a perfect impedance match in the vicinity of one of its ports, the network equivalent circuit may have a nonzero susceptance at that port, depending upon internal structure of actual network.

As stated above, intrinsic insertion loss can be achieved by adding proper port susceptances which will yield bilateral network match. Experimentally, this match can be obtained without solving for all of the equivalent circuit constants. First, the proper susceptance added at the input port will yield $S_{\max} = S_{\min}$. Then the proper susceptance at the output port will result in an admittance locus whose iconocenter will be at the center of the Smith chart.

The concept of intrinsic insertion loss can be extended to networks where the port transmission lines differ in characteristic impedances as well as in propagating modes by using a reflectionless tapered attenuator α_0 in the equivalent circuit. In this case, b_1 and b_2 are normalized to the characteristic impedances of the respective port lines.

The applicability of intrinsic insertion loss may appear to be somewhat dubious when analyzing a simple microwave network containing either a lumped shunt or series resistance. For certain positions of the adjustable short circuit, the input VSWR becomes infinite, resulting in zero-db intrinsic loss, according to (6) or (7). In the limit, the zero loss can be achieved by using an infinite susceptance near either port, which will place the shunt resistance at a voltage minimum or the series resistance at a voltage maximum. For the same simple circuit with infinite $S_{\max}$, α_{net} is finite, however, in accordance with (4).

Acknowledgment

The author wishes to thank Dr. Seymour B. Cohn of the Stanford Research Institute and Dr. Seymour Edelberg of Balco Research Laboratories for valuable discussions on lossy microwave networks.

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Klystron Noise

In my paper "Crystal Checker for Balanced Mixers"[1] I gave data on the excess noise of typical klystrons. Since that paper was prepared, further data has been obtained that permits an expansion of Fig. 6.

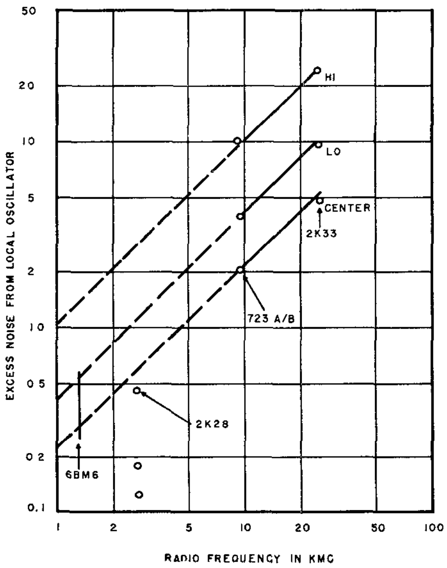

Fig. 1—30-mc excess noise of typical klystrons. (Same as Fig. 6 of original paper except for addition of 2K28 data.)

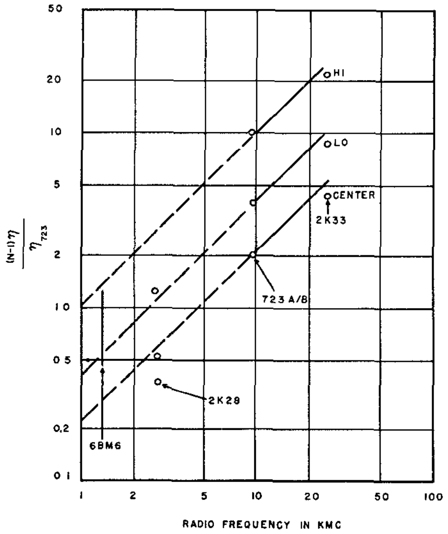

Fig. 2—30-mc excess noise of typical klystrons multiplied by the efficiency ratio η/η_{723}.

Measurements were made on a 2K28 at 2800 mc. Fig. 1 is a revision of Fig. 6 to show the new data. It is seen that the new points fall below the interpolated curves given originally, although the relative vertical spacings are about the same as those predicted by interpolation. The data in Fig. 1 seem to fit the empirical relationship

$$N - 1 \cong K \frac{f}{\eta},$$

where $N-1$ is the excess noise power at a particular intermediate frequency, K is a constant, and η is the efficiency of converting beam power to cw power.

If one arbitrarily multiplies the data in Fig. 1 by the ratio of the efficiency of the particular klystron to that of the 723A/B, the nearly linear relationship of Fig. 2 is obtained.

If the empirical relation is valid, Fig. 2 can be used to predict the approximate performance of other klystrons by spotting the operating frequency on the figure, or an extension thereof, and multiplying by the ratio of efficiencies, η_{723}/η.

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[1] *Trans. I.R.E.*, vol. MTT-2, pp. 10–15; July, 1954.

A Practical Method of Locating Waveguide Discontinuities

In the maintenance of ultra-high frequency equipments utilizing waveguide, one of the more difficult troubles to diagnose and correct is that of the high standing wave ratio. The question always arises as to whether the antenna or the waveguide is at fault. Sometimes a visual inspection of the transmission system will disclose the difficulty. Too often, however, the physical layout of the system makes a close inspection impractical. The construction of many antennas, also, does not permit an examination of the radio frequency components without a complex mechanical disassembly, costly in terms of man-hours.

It is desirable, therefore, to determine the approximate location of the discontinuity electronically. On those equipments having a continuously variable-frequency transmitter, a frequency measuring device, and a waveguide probe for sampling the standing wave, this can be done quite easily. On other equipments these features can be simulated by installing, at the transmitter end of the line, a waveguide section equipped with probes for inserting the signal of a variable-frequency, calibrated, test oscillator and for sampling the standing wave.

The usual analysis of a standing wave requires the movement of the probe along the slotted waveguide; the detected voltage progresses through maximum and minimum values in accordance with the standing wave pattern.

If, however, the probe is left at a fixed position and the frequency is varied, the standing wave will move past the probe, its detected voltage rising and falling in the same manner as the *guide* wavelength changes with frequency. It will be shown that the frequency change necessary to move the standing wave a specific number of wavelengths is a function of the distance from the probe to the discontinuity causing the standing wave.

If we let N equal the number of half-guide wavelengths between the probe and the discontinuity, and let L represent the physical distance from the probe to the discontinuity, then

$$N = \frac{2L}{\lambda_g}. \tag{1}$$

Now, if the operating frequency is increased sufficiently to bring one more half-guide wavelength into the distance, L, then

$$N + 1 = \frac{2L}{\lambda'_g}. \tag{2}$$

Subtracting (1) from (2) and rearranging, we have

$$L = \frac{\lambda_g \lambda'_g}{2(\lambda_g - \lambda'_g)}. \tag{3}$$

In making use of this phenomenon to locate a serious discontinuity in a waveguide transmission system, we must determine the guide wavelengths that will give us two successive maxima (or minima) of the standing wave at a fixed probe location, as the frequency is varied.

The guide wavelength is a function of frequency which can be evaluated from the identity

$$\lambda_g = \frac{c}{\sqrt{f^2 - \left(\frac{c}{2b}\right)^2}}, \tag{4}$$

in which

c is free space velocity of propagation,
f is operating frequency,
b is wide inside dimension of the waveguide.

Thus, we are approaching a practical solution to the problem, since the frequencies required to give two successive maxima (or minima) of the standing wave are measurable. Once the frequencies are determined, they are converted to wavelengths in (4); the wavelengths, in turn, are used in (3) to give the distance from the probe to the discontinuity.

In practice, frequency is measured with a calibrated echo box or wave meter. Standing wave voltage is measured with a vacuum tube volt meter equipped with radio frequency probe. (Calibration of this instrument is not necessary since only relative readings of voltage are required to establish the maximum and minimum positions of the standing wave.) If possible, the detector of the vacuum tube voltmeter should be con-

(*Cont'd on p. 46*)